Exact deflection of a Neutral-Tachyon in the Kerr's Gravitational field.

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October 7, 2011

Abstract

We solve in closed analytic form space-like geodesic equations in the Kerr gravitational field. Such geodesic equations describe the motion of neutral tachyons (faster than light particles) in the Kerr spacetime. More specifically we derive the closed form solution for the deflection angle of a neutral tachyon on an equatorial orbit in Kerr spacetime. The solution is expressed elegantly in terms of Lauricella's hypergeometric function F_D.We applied our results to three cases: first, for the calculation of the deflection angle of a neutral tachyon on an equatorial trajectory in the gravitational field of a Kerr black hole. Subsequently, we applied our exact solutions to compute the deflection angle of equatorial spacelike geodesics in the gravitational fields of Sun and Earth assuming the Kerr spacetime geometry.

1 Introduction

Recent results from the OPERA experiment, taken at face value, seem to indicate the existence of superluminal neutrino species, whose speed υ as it has been determined by OPERA collaboration with respect to the speed of light, c , is [1]:

$$\frac{\upsilon - c}{c} = (2.48 \pm 0.28(\text{stat}) \pm 0.30(\text{sys})) \times 10^{-5} \tag{1}$$

for a mean energy of the neutrino beam of 17GeV[1].

This result, if it will stand to further scrutiny and will be confirmed by independent measurements it will certainly constitute a fundamental discovery about Nature.

For us it means, that we are endowed with sufficient motivation to explore further properties and aspects of the general hypothesis of the existence of superluminal (tachyon) particles in fundamental physics. In particular, we intend in this Letter to explore the interaction of a neutral tachyon field with the strong gravitational field of Kerr spacetime. We shall obtain the exact analytic solution for the deflection angle of a neutral tachyon in an equatorial unbound orbit in Kerr spacetime.

The plausible existence of a superluminal particle in the framework of special relativity (SR) has been discussed by various authors [2] (see also [3]). The energy of a tachyon particle is given by [4]

$$E = \frac{|m|c^2}{\sqrt{\frac{\upsilon^2}{c^2} - 1}}, \text{ for } \upsilon > c \tag{2}$$

where $|m|$ denotes the magnitude [1] of the tachyon's imaginary rest mass: $m = i|m|$. The energy momentum vector is now

$$p^\mu = \left(\frac{|m|c}{\sqrt{\frac{\upsilon^2}{c^2} - 1}}, \frac{|m|}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \frac{\mathrm{d}x}{\mathrm{d}t}, \frac{|m|}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \frac{\mathrm{d}y}{\mathrm{d}t}, \frac{|m|}{\sqrt{\frac{\upsilon^2}{c^2} - 1}} \frac{\mathrm{d}z}{\mathrm{d}t} \right), \text{ for } \upsilon > c \tag{3}$$

and the dispersion relation is valid

$$p^\mu p_\mu = (E/c)^2 - p_x^2 - p_y^2 - p_z^2 = -|m|c^2. \tag{4}$$

Assuming the gravitational mass of the tachyon is exactly equal to the magnitude of its imaginary rest mass the tachyon moves along spacelike geodesics in a gravitational field. Thus since spacelike geodesics are part of the

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[1]Sometimes refer to as the *metamass* [2].

theory of General Relativity (GTR), tachyons can be considered as implicit ingredients of the theory if the above assumption is valid. We mention at this point earlier works on this matter. The author of [5] has performed a weak field calculation of the deflection angle of a neutral tachyon in the gravitational field of the Sun assuming a Schwarzschild spacetime geometry. Tachyons in uniform relativistic cosmology have been considered in [6] and an initial study of the conditions for tachyons to be captured by a Kerr black hole has been performed in [7]. However, an exact analytic calculation of the deflection angle for an equatorial orbit in the Kerr gravitational field is still lacking. Our Letter fills this important gap in the field. We start our discussion with the derivation of spacelike geodesics in the Kerr spacetime with a cosmological constant present and subsequently we obtain the closed form analytic solution for the deflection angle of an unbound tachyonic equatorial orbit in the Kerr gravitational field . The exact solution is expressed in terms of generalized hypergeometric functions of Appell-Lauricella. We apply our closed form solutions to three cases. We determine the deflection angle that an equatorial spacelike geodesic undergoes in the gravitational fields of the Sun and Earth assuming a Kerr geometry for modelling these fields. We also apply our solutions to compute the exact deflection angle that a neutral tachyon on an equatorial trajectory undergoes in the gravitational field exerted by a rotating (Kerr) black hole. We leave the last section for our conclusions.

2 Spacelike geodesics in Kerr-de Sitter spacetime.

Taking into account the contribution from the cosmological constant Λ, the generalization of the Kerr solution [8] is described by the Kerr-de Sitter metric element which in Boyer-Lindquist (BL) coordinates is given by [9]-[10]:

$$\begin{aligned} \mathrm{d}s^2 = &\frac{\Delta_r}{\Xi^2\rho^2}(c\mathrm{d}t - a\sin^2\theta\mathrm{d}\phi)^2 - \frac{\rho^2}{\Delta_r}\mathrm{d}r^2 - \frac{\rho^2}{\Delta_\theta}\mathrm{d}\theta^2 \\ &- \frac{\Delta_\theta \sin^2\theta}{\Xi^2\rho^2}(ac\mathrm{d}t - (r^2+a^2)\mathrm{d}\phi)^2 \end{aligned} \tag{5}$$

$$\Delta_\theta := 1 + \frac{a^2\Lambda}{3}\cos^2\theta, \quad \Xi := 1 + \frac{a^2\Lambda}{3} \tag{6}$$

$$\Delta_r := \left(1 - \frac{\Lambda}{3}r^2\right)(r^2+a^2) - 2\frac{GM}{c^2}r, \quad \rho^2 = r^2 + a^2\cos^2\theta \tag{7}$$

We denote by a the rotation (Kerr) parameter and M denotes the mass of the spinning black hole. Choosing a real affine parameter λ for the spacelike geodesic by $\mathrm{d}\lambda^2 = -\mathrm{d}s^2$, we have the geodesic equation in the usual notation as

$$\frac{\mathrm{d}^2x^i}{\mathrm{d}\lambda^2} + \Gamma^i_{jk}\frac{\mathrm{d}x^j}{\mathrm{d}\lambda}\frac{\mathrm{d}x^k}{\mathrm{d}\lambda} = 0, \tag{8}$$

where x^i denote the BL coordinates and Γ^i_{jk} the Christoffel symbols of the second kind.

The geodesic equations in Kerr spacetime in the presence of the cosmological constant Λ were derived in [11] by solving the Hamilton-Jacobi differential equations by separation of variables. The tachyon motion, as we mentioned earlier, is described by the spacelike geodesics (which are the first integrals of (8)) which take the form[2]:

$$\begin{aligned} \int \frac{\mathrm{d}r}{\sqrt{R}} &= \int \frac{\mathrm{d}\theta}{\sqrt{\Theta}}, \\ \rho^2\frac{\mathrm{d}\phi}{\mathrm{d}\lambda} &= -\frac{\Xi^2}{\Delta_\theta\sin^2\theta}(aE\sin^2\theta - L) + \frac{a\Xi^2}{\Delta_r}[(r^2+a^2)E - aL] \\ c\rho^2\frac{\mathrm{d}t}{\mathrm{d}\lambda} &= \frac{\Xi^2(r^2+a^2)[(r^2+a^2)E-aL]}{\Delta_r} - \frac{a\Xi^2(aE\sin^2\theta - L)}{\Delta_\theta} \\ \rho^2\frac{\mathrm{d}r}{\mathrm{d}\lambda} &= \pm\sqrt{R} \\ \rho^2\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} &= \pm\sqrt{\Theta} \end{aligned} \tag{9}$$

[2]The tachyon metamass $|m|$, which is one of the first integrals of (8), was set equal to unity without loss of generality.

where

$$
\begin{aligned}
R &:= \Xi^2[(r^2+a^2)E-aL]^2-\Delta_r(-r^2+Q+\Xi^2(L-aE)^2) \\
\Theta &:= [Q+(L-aE)^2\Xi^2+a^2\cos^2\theta]\Delta_\theta-\Xi^2\frac{(aE\sin^2\theta-L)^2}{\sin^2\theta}
\end{aligned}
\tag{10}
$$

The constants of motion E, L are associated with the isometries[3] of the Kerr metric while Q denotes Carter's constant, the fourth constant of integration. The spacelike Kerr geodesics are obtained by setting $\Lambda = 0$ in Eqs.(9)-(10).

3 Exact solution of equatorial spacelike geodesics in Kerr spacetime

We now proceed to determine the exact solution for the deflection of a neutral tachyon in an equatorial spacelike orbit in Kerr spacetime assuming $\Lambda = 0$. The more general case with the cosmological constant present will be a subject of a separate publication [15]. We have $r^2(\dot{r}) = \sqrt{R}$.This can be rewritten as [4]

$$
\dot{r}^2 = E^2+\frac{a^2E^2}{r^2}-\frac{L^2}{r^2}+\frac{2GM}{c^2r^3}(L-aE)^2+\frac{\Delta}{r^2}. \tag{11}
$$

where Δ is obtained by setting $\Lambda = 0$ in equation (7) for Δ_r. By defining a new variable $u := 1/r$ we obtain the following expression:

$$
u^{-4}\dot{u}^2 = E^2+a^2E^2u^2-L^2u^2+\frac{2GM}{c^2}u^3(L-aE)^2+(1+a^2u^2-\frac{2GM}{c^2}u) \equiv B_{tac}(u). \tag{12}
$$

Similarly the geodesic for the azimuth coordinate is given

$$
\dot{\phi}^2 = u^4\frac{A^2(u)}{D^2(u)} \tag{13}
$$

where

$$
A(u) := L+\alpha_S u(aE-L), \qquad D(u) := 1+a^2u^2-\alpha_S u, \qquad \alpha_S := \frac{2GM}{c^2}. \tag{14}
$$

Thus we derive the orbital equation

$$
\frac{\mathrm{d}\phi}{\mathrm{d}u} = \frac{A(u)}{D(u)}\frac{1}{\sqrt{B_{tac}(u)}}. \tag{15}
$$

We now use the technique of partial fractions from integral calculus in order to calculate the deflection of the neutral tachyon's equatorial orbit in the Kerr gravitational field from equation (15). We write:

$$
\frac{A(u)}{D(u)} = \frac{A_+}{u_+-u}+\frac{A_-}{u_--u} \tag{16}
$$

where $u_+ = \frac{r_+}{a^2}, u_- = \frac{r_-}{a^2}$ and

$$
r_\pm = \frac{GM}{c^2}\pm\sqrt{\left(\frac{GM}{c^2}\right)^2-a^2} \tag{17}
$$

denote the radii of the event and Cauchy horizons respectively for the case of a Kerr black hole. Also the quantities A_+, A_- are given by

$$
A_+ = \frac{\frac{L}{a^2}+\frac{\alpha_S}{a^2}(aE-L)u_+}{u_--u_+} \qquad A_- = \frac{\frac{-L}{a^2}-\frac{\alpha_S}{a^2}(aE-L)u_-}{u_--u_+} \tag{18}
$$

In order to calculate the angle of deflection for the tachyon it is necessary to calculate the integral: $\Delta\phi^{GTR}_{Tachyon} = 2\int_0^{u'_2}\mathrm{d}\phi$.Using the formalism developed in references [11]-[13] , for computing hyperelliptic integrals in closed

[3]i.e. they are related to the energy and angular momentum per unit metamass of the tachyon at infinity.
[4]For equatorial geodesics $\theta = \pi/2$, $Q = 0$.

analytic form, in terms of Lauricella's hypergeometric function F_D , we compute:

$$\begin{aligned}
\Delta\phi^{GTR}_{Tachyon} = & \frac{2}{\sqrt{u_1'-u_3'}}\frac{1}{\sqrt{\frac{\alpha_S(L-aE)^2}{(\frac{GM}{c^2})^3}}}\Bigg\{\frac{A_+}{\frac{GMr_+}{c^2a^2}-u_3'}\left(F_1\left(\frac{1}{2},\beta_A,1,\mathbf{z}^{r_+}_A\right)\pi\right.\\
& \left.-2\sqrt{\frac{-u_3'}{u_2'-u_3'}}F_D\left(\frac{1}{2},\beta^4_3,\frac{3}{2},\mathbf{z}^{r_+}_D\right)\right)\\
& +\frac{A_-}{\frac{GMr_-}{c^2a^2}-u_3'}\left(F_1\left(\frac{1}{2},\beta_A,1,\mathbf{z}^{r_-}_A\right)\pi\right.\\
& \left.-2\sqrt{\frac{-u_3'}{u_2'-u_3'}}F_D\left(\frac{1}{2},\beta^4_3,\frac{3}{2},\mathbf{z}^{r_-}_D\right)\right)\Bigg\}
\end{aligned} \tag{19}$$

where

$$\beta_A=\left(1,\frac{1}{2}\right),\quad \beta^4_3=\left(1,\frac{1}{2},\frac{1}{2}\right) \tag{20}$$

and

$$\mathbf{z}^{r_\pm}_A=\left(\frac{u_2'-u_3'}{\frac{GMr_\pm}{c^2a^2}-u_3'},\frac{u_2'-u_3'}{u_1'-u_3'}\right),\quad \mathbf{z}^{r_\pm}_D=\left(\frac{-u_3'}{\frac{GMr_\pm}{c^2a^2}-u_3'},\frac{-u_3'}{u_1'-u_3'},\frac{-u_3'}{u_2'-u_3'}\right) \tag{21}$$

We have also defined: $u'=u\frac{GM}{c^2}$. The roots of the cubic in this case are real and organized in the ascending order

$$u_1'>u_2'>0>u_3' \tag{22}$$

The angle of deflection δ of a neutral tachyon equatorial trajectory from the gravitational field of a rotating black hole or a rotating central mass is defined to be the deviation of $\Delta\phi^{GTR}_{Tachyon}$ from the transcendental number π

$$\delta=\Delta\phi^{GTR}_{Tachyon}-\pi. \tag{23}$$

We shall apply our closed form solution, Eq.(19) for the deflection angle of an equatorial neutral tachyon's orbit in the gravitational field of Kerr spacetime in two cases. First, we shall compute the deflection angle δ, for a tachyon in the gravitational field of the Sun (assuming the Kerr spacetime geometry). Second, the deflection angle of a neutral tachyon in an equatorial orbit around a Kerr black hole for various values of the involved physical parameters. The physical parameters are the velocity of the tachyon particle υ (at large distances from the central mass), the energy per unit mass (in units of c^2) $E=\frac{1}{\sqrt{\upsilon^2/c^2-1}}$, the parameter L and the spin a (Kerr parameter) of the black hole.

3.1 Deflection of neutral tachyon in an equatorial orbit in the Kerr (Sun's) gravitational field.

Assuming that the gravitational field of the spinning Sun is described by the Kerr spacetime geometry we compute the deflection angle of a tachyon in an unbound equatorial orbit using our closed form solution Eq.(23). We repeat the calculation for three values of the velocity of the tachyon particle. Namely: 1) $\upsilon=(1+2.48\times10^{-5})c$ (same as OPERA's experimental value) 2) $\upsilon=\sqrt{2}c$ 3) $\upsilon=10^6c$. For the parameter E we use the formula $E=\frac{1}{\sqrt{\upsilon^2/c^2-1}}$. Taking the point of closest approach $r_0=R_\odot=6.9551\times10^8\,\mathrm{m}$ the parameter L is chosen to have the value: $L=471013.2781620\upsilon E\frac{G_NM_\odot}{c^2}$.Our results are summarized in Table 1. We note that the deflection angle ranges from half the value of a photon's deflection from the Sun's gravitational field (case 3 high velocitites, low energies for the tachyon) to a value equal to the deflection of light for the case 1) i.e. tachyon velocity as the one determined by OPERA experiment (1).

Our findings are consistent with the results of [5], in which, a perturbative calculation of the deflection angle for a tachyon in a Schwarzschild field of the Sun was performed.

3.2 Deflection of a neutral tachyon in an equatorial orbit in the Kerr black hole spacetime

Again for different choices of values for the velocity of a tachyon we compute the deflection angle for different values for the spin of the black hole and the remaining parameters. For the three different values of the tachyon particle we choose first $L=100\upsilon E\frac{GM_{\rm BH}}{c^2}$. We present our results in table 2.

We repeat the analysis for $L=40\upsilon E\frac{GM_{\rm BH}}{c^2}$. Our results are dislayed in Table 3.

Case1: $\upsilon=(1+2.48\times 10^{-5})c$	Case 2: $\upsilon=\sqrt{2}c$	Case 3: $\upsilon=10^6 c$
$\delta=1.75164$arcsec	$\delta=1.31376$arcsec	$\delta=0.875837$arcsec

Table 1: Deflection of a neutral tachyon's equatorial trajectory in the gravitational field of the Sum assuming Kerr spacetime geometry. The value of the Kerr parameter was chosen to be $a=0.2158\frac{GM_\odot}{c^2}$.

a	$\upsilon=(1+2.48\times 10^{-5})c$	$\upsilon=\sqrt{2}c$	$\upsilon=10^6 c$
0.52	$\delta=0.04099735=8456.31$ arcsec	$\delta=0.0305724=6306.01$ arcsec	$\delta=0.0202396=4174.71$ arcsec
0.99616	$\delta=0.04079425=8414.42$ arcsec	$\delta=0.0304312=6276.89$ arcsec	$\delta=0.02024108=4175.02$ arcsec

Table 2: Deflection angle of a neutral tachyon in an equatorial orbit in the gravitational field of a Kerr black hole. We present our results for three different values of the velocity of the tachyon particle and two choices for the spin of the black hole. Also $L=100\upsilon E\frac{GM_{\rm BH}}{c^2}$ and $E=\frac{1}{\sqrt{\upsilon^2/c^2-1}}$.

3.3 Deflection of a neutral tachyon in an equatorial orbit by the gravitational field of Earth

Assuming the Earth's gravitational field is described by a Kerr spacetime, we shall compute the deflection angle of a neutral tachyon in an equatorial orbit in Earth's gravitation. There is a further novelty in this calculation. The Kerr parameter that corresponds to the angular momentum of Earth is equal to $a_\oplus=329.432\,\text{cm}=371.398(2GM_\oplus/c^2)$[14]. Thus the two roots of the polynomial $D(u)$ are complex-conjugate. Therefore, when we calculate exactly the hyperelliptic integral, two of the variables of Lauricella's function F_D are complex-conjugates. This is fine as long as their modules are less than 1,which indeed it is the case in our computations.

Thus

$$\begin{aligned}\int \mathrm{d}\phi &= 2\int_0^{u_2'}\frac{A(u)}{D(u)}\frac{1}{\sqrt{B_{tac}(u)}}\mathrm{d}u\\ &= 2\int_0^{u_2'}\frac{\mathrm{d}u' L}{a^2\left(\frac{GM_\oplus r_+}{c^2a^2}-u'\right)\left(\frac{GM_\oplus r_-}{c^2a^2}-u'\right)}\frac{1}{\sqrt{\frac{\alpha_S(L-aE)^2}{(GM_\oplus/c^2)^3}}}\frac{1}{\sqrt{(u'-u_3')(u_1'-u')(u_2'-u')}}+\\ &2\int_0^{u_2'}\frac{\alpha_S(aE-L)\mathrm{d}u'u'}{a^2\left(\frac{GM_\oplus r_+}{c^2a^2}-u'\right)\left(\frac{GM_\oplus r_-}{c^2a^2}-u'\right)\sqrt{\frac{\alpha_S(L-aE)^2}{(GM_\oplus/c^2)^3}}\sqrt{(u'-u_3')(u_1'-u')(u_2'-u')}}\end{aligned} \tag{24}$$

Now applying the transformation

$$\boxed{u'=u_2'(1-t),}$$

we have:

$$\frac{GM_\oplus r_\pm}{c^2a^2}-u'=\left(\frac{GM_\oplus r_\pm}{c^2a^2}-u_2'\right)\left[1+\frac{tu_2'}{\frac{GM_\oplus r_\pm}{c^2a^2}-u_2'}\right],\quad u_2'-u'=u_2't, \tag{25}$$

$$u_1'-u'=(u_1'-u_2')\left[1+\frac{u_2't}{u_1'-u_2'}\right],\quad u'-u_3'=(u_2'-u_3')\left[1-\frac{u_2't}{u_2'-u_3'}\right] \tag{26}$$

thus we tranform our integral onto the integral representation of Lauricella's function F_D of four-variables (see also

a	$\upsilon=(1+2.48\times 10^{-5})c$	$\upsilon=\sqrt{2}c$	$\upsilon=10^6 c$
0.52	$\delta=0.10652474=21972.3$ arcsec	$\delta=0.0787036=16233.8$ arcsec	$\delta=0.05153626=10630.1$ arcsec
0.99616	$\delta=0.10512217=21683$ arcsec	$\delta=0.0777545=16038$ arcsec	$\delta=0.05156155=10635.3$ arcsec

Table 3: Deflection angle of a neutral tachyon in an equatorial orbit in the gravitational field of a Kerr black hole. We present our results for three different values of the velocity of the tachyon particle and two choices for the spin of the black hole. Also $L=40\upsilon E\frac{GM_{\rm BH}}{c^2}$ and $E=\frac{1}{\sqrt{\upsilon^2/c^2-1}}$.

Appendix):

$$\int \mathrm{d}\phi = \Delta\phi^{GTR}_{NT\oplus}$$
$$= \frac{2}{a^2\left(\frac{GM_\oplus r_+}{c^2a^2}-u_2'\right)\left(\frac{GM_\oplus r_-}{c^2a^2}-u_2'\right)\sqrt{u_2'(u_2'-u_3')(u_1'-u_2')}\sqrt{\frac{\alpha_S(L-aE)^2}{(GM_\oplus/c^2)^3}}}\times$$
$$\Bigg[\alpha_S(aE-L)u_2'^2\frac{\Gamma(1/2)\Gamma(2)}{\Gamma(5/2)}F_D\left(\frac{1}{2},\beta_4^1,\frac{5}{2},\mathbf{z}_r^\oplus\right)+$$
$$Lu_2'\frac{\Gamma(1/2)\Gamma(1)}{\Gamma(3/2)}F_D\left(\frac{1}{2},\beta_4^1,\frac{3}{2},\mathbf{z}_r^\oplus\right)\Bigg] \tag{27}$$

where

$$\beta_4^1=\left(1,1,\frac{1}{2},\frac{1}{2}\right),\quad \mathbf{z}_r^\oplus=\left(\frac{-u_2'}{\frac{GM_\oplus r_+}{c^2a^2}-u_2'},\frac{-u_2'}{\frac{GM_\oplus r_-}{c^2a^2}-u_2'},\frac{u_2'}{u_2'-u_3'},\frac{-u_2'}{u_1'-u_2'}\right) \tag{28}$$

We apply our analytic solution eq.(27) for computing the deflection angle of a neutral tachyon in the gravitational field of Earth. We choose as value for the velocity of the tachyon the central value of OPERA experiment $\upsilon = (1+2.48\times10^{-5})c$. Taking the point of closest approach as the radius (equatorial) of Earth $r_0 = R_\oplus = 6.378137\times10^6\,\mathrm{m}$ we have that the parameter $L = \upsilon E\times 1.438127796068894\times10^9$. For the Kerr parameter of Earth we choose the above mentioned value. Then we determine $\delta = \Delta\phi^{GTR}_{NT\oplus}-\pi = 2.78\times10^{-9}\,\mathrm{rad}\sim 0.000573\,\mathrm{arcsec}$.

4 Conclusions

In this paper we investigated tachyon orbits (spacelike geodesics) in Kerr spacetime. More specifically, we derived the closed form solution for the deflection angle of a neutral tachyon in an equatorial orbit in the gravitational field of Kerr spacetime. The solution was expressed elegantly in terms of Lauricella's multivariable hypergeometric function F_D.We applied our exact solutions in three cases: 1) we calculated the deflection of a neutral tachyon by the gravitational field of a rotating Kerr black hole, for different values of the velocity of the tachyon and the spin of the black hole. We note the strong dependence of the deflection angle on the spin of the spinning black hole for low tachyon velocities, especially for lower values for the parameter L. Large magnitudes for the deflection angle were produced see table 3. 2) we calculated the deflection of equatorial neutral tachyon orbits by the gravitational field of our Sun assuming a curved Kerr spacetime geometry. For low tachyon velocities (such as the ones reported by the OPERA collaboration) the deflection angle was calculated to have a value $\sim 1.75\,\mathrm{arcsec}$ a value equals to the amount of deflection that light experiences by the gravitation field of our Solar system star. For high tachyon velocities $\upsilon \gg c$ the calculated deflection angle decreases to half the value of δ at low superluminal velocities. 3) we calculated the deflection angle of an equatorial tachyon trajectory in the gravitational field of Earth assuming a Kerr geometry. There is a further novelty in the calculation. The solution for δ is expressed in terms of Lauricella's hypergeometric function F_D of four variables two of which are complex-conjugates. The neutral tachyon undergoes a small deflection of 2.78×10^{-9} radians$\sim 0.000573\,\mathrm{arcsec}$. Thus if tachyons do exist and move on spacelike geodesics they undergo a deflection by the gravitational field of the rotating central mass. The deflection exhibits a strong dependence on the superluminal velocity and the spin of the rotating mass. This gravitational effect is in principle measurable. It will be interesting to generalize our results to the case of finding the exact solutions for generic non-equatorial spacelike orbits in the presence of the cosmological constant Λ (spacelike non-equatorial Kerr-de Sitter orbits). However, such an investigation is beyond the scope of the current paper and it will be the subject of a separate publication [15].

Acknowledgement 1 *The author acknowledges useful discussions with Giota Grigoriadou.*

5 Appendix

We introduce the Lauricella function F_D and its integral representation

$$\boxed{F_D(\alpha,\beta,\gamma,\mathbf{z})=\sum_{n_1,n_2,\ldots,n_m=0}^{\infty}\frac{(\alpha)_{n_1+\cdots n_m}(\beta_1)_{n_1}\cdots(\beta_m)_{n_m}}{(\gamma)_{n_1+\cdots+n_m}(1)_{n_1}\cdots(1)_{n_m}}z_1^{n_1}\cdots z_m^{n_m}} \tag{29}$$

where

$$\begin{aligned}\mathbf{z} &= (z_1, \ldots, z_m),\\ \beta &= (\beta_1, \ldots, \beta_m).\end{aligned} \tag{30}$$

The Pochhammer symbol $\boxed{(\alpha)_m = (\alpha, m)}$ is defined by

$$(\alpha)_m = \frac{\Gamma(\alpha+m)}{\Gamma(\alpha)} = \begin{cases} 1, & \text{if} \quad m=0 \\ \alpha(\alpha+1)\cdots(\alpha+m-1) & \text{if} \quad m=1,2,3 \end{cases} \tag{31}$$

With the notations $\mathbf{z}^{\mathbf{n}} := z_1^n \ldots z_m^n$, $(\beta)_{\mathbf{n}} := (\beta_1)_{n_1} \ldots (\beta_m)_{n_m}$, $\mathbf{n}! = n_1! \ldots n_m!, |\mathbf{n}| := n_1 + \ldots n_m$ for m-tuples of numbers in (30) and of non-negative integers $\mathbf{n} = (n_1, \ldots n_m)$ the Lauricella series F_D in compact form is

$$F_D(\alpha, \beta, \gamma, \mathbf{z}) := \sum_{\mathbf{n}} \frac{(\alpha)_{|\mathbf{n}|}(\beta)_{\mathbf{n}}}{(\gamma)_{|\mathbf{n}|}\mathbf{n}!}\mathbf{z}^{\mathbf{n}}. \tag{32}$$

The series admits the following integral representation:

$$\boxed{F_D(\alpha, \beta, \gamma, \mathbf{z}) = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \int_0^1 t^{\alpha-1}(1-t)^{\gamma-\alpha-1}(1-z_1 t)^{-\beta_1} \cdots (1-z_m t)^{-\beta_m} \mathrm{d}t} \tag{33}$$

which is valid for $\boxed{\mathrm{Re}(\alpha) > 0,\ \mathrm{Re}(\gamma-\alpha) > 0}$. It *converges absolutely* inside the m-dimensional cuboid:

$$|z_j| < 1, (j = 1, \ldots, m). \tag{34}$$

For $m = 2$, F_D becomes Appell's F_1 two variable hypergeometric function $F_1(\alpha, \beta, \beta', \gamma, x, y)$ with integral representation

$$\int_0^1 u^{\alpha-1}(1-u)^{\gamma-\alpha-1}(1-ux)^{-\beta}(1-yu)^{-\beta'}\ \mathrm{d}u = \frac{\Gamma(\alpha)\Gamma(\gamma-\alpha)}{\Gamma(\gamma)} F_1(\alpha, \beta, \beta', \gamma, x, y). \tag{35}$$

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